Flow in rocks modelled as multiphase continua: application to polymineralic rocks

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Abstract—The notion of a multiphase continuum is applied to the problem of determining the flow properties of two-phase rocks. The assumptions made include (i) that both the bulk rock and the constituent phases have a linear viscous rheology, (ii) that the phases are uniformly mixed and distributed in the rocks, and (iii) that no segregation occurs during the deformation. A theoretical analysis is made assuming the additive relationships for the linear momentum, the stresses and the entropy production rates. Two possible relationships between the normalized bulk rock viscosity (the ratio of bulk rock viscosity to the viscosity of the less viscous phase) and the volume fraction of the constituent phase are derived. The first relationship is linear (mode 1 behaviour), while the second is non-linear (mode 2 behaviour). In mode 2 behaviour, the normalized bulk rock viscosity decreases at first rapidly and then less so as the volume fraction of the less viscous phase increases from zero. The rate of the initial rapid decrease in viscosity is in proportion to the viscosity contrast between two phases and in inverse proportion to the density contrast. The mode 2 behaviour shows good agreement with published deformation experiments of synthetic two-phase rocks. The bulk rock deformation rate can be related to those of the constituent phases. In mode 1 behaviour there is no deformation rate partitioning. In contrast, in mode 2 behaviour the ratio of the deformation rate partition coefficients of the less viscous phase to that of the more viscous phase is equal to the ratio of the inverse viscosity contrast to the density contrast. This relationship corresponds to the rule of strain refraction derived for multilayer systems. Mode 2 behaviour can be further subdivided into two regimes: one where the viscosity contrast is greater than the density contrast (mode 2a), and a second where the opposite is true (mode 2b). In mode 2a behaviour more strain is partitioned into the less viscous phase; while, in mode 2b behaviour more strain is partitioned into the more viscous phase. © 1998 Elsevier Science Ltd. All rights reserved

INTRODUCTION

Almost all rocks in the crust and the mantle are polymineralic. Knowledge of the mechanical (and kinematic) flow behaviour of multiphase materials is, therefore, important in understanding mechanical states and dynamic behaviour of the lithosphere and asthenosphere. In addition, this information is useful for structural geologists and petrologists trying to reconstruct the original state of rocks using their knowledge of the final state and the physical properties of rocks. Much effort has been made to understand the rheological properties of polymineralic rocks in terms of their constituent phases and the microstructures (Gay, 1968; Bilby *et al.*, 1975; Price, 1982; Tharp, 1983; Ross *et al.*, 1987; Jordan, 1987, 1988; Handy, 1990, 1992, 1994; Tullis *et al.*, 1991; Ji and Zhao, 1993, 1994; Bloomfield and Covey-Crump, 1993; Dell'Angelo and Tullis, 1996; Zhao and Ji, 1997). The flow strength of rocks that are composed of two solid phases has been investigated experimentally (Price, 1982; Jordan, 1987, 1988; Ross *et al.*, 1987; Bloomfield and Covey-Crump, 1993) and theoretically (Gay, 1968; Tharp, 1983; Jordan, 1988; Handy, 1990, 1994; Tullis *et al.*, 1991; Ji and Zhao, 1993, 1994; Zhao and Ji, 1997).

An important observation from experimental work is the presence of an abrupt drop in strength with only a small addition of a weaker phases (e.g. Price, 1982;

Jordan, 1987; Ross *et al.*, 1987). For example, in deformation experiments of synthetic calcite–halite aggregates, Jordan (1987, 1988) observed an abrupt drop in strength with a small addition of the weaker halite phase and a subsequent further gentle decrease with increasing volume fraction of halite. Jordan (1988) ascribed this phenomenon to a discontinuous microstructural change at intermediate compositions, from a state of a load bearing framework in which almost all the load is supported by the stronger phase, to a state which is described by a two-block model and in which almost all the strain is accommodated by the weaker phase. Strictly speaking, however, the model Jordan (1988) used for lower volume fractions of the less viscous phase is only valid when one phase is inviscid material. Moreover the application of the model is limited to the case of large viscosity contrasts between the two phases and much lower volume fractions of the less viscous phase (e.g. Bloomfield and Covey-Crump, 1993; Ji and Zhao, 1994). Zhao and Ji (1997) presented a model that predicts the flow strength of two-phase rocks on the basis of the fibre-loading model. A good agreement is obtained between the theory and the experimental observation. The fibre-loading model is however, limited to low concentrations of the 'fibre' material, and its application is limited to the case of uniaxial deformation.

It is clear that the physical properties of composite materials will be related to those of the constituent

phases not only in terms of mechanical behaviour but also kinematic behaviour. In particular, an important question is how the deformation rate of the composite rock is related to deformation rate of its constituent phases. The bulk deformation rate is likely to be partitioned between its constituent phases. Some structural geologists use the term 'partitioning' to indicate spatial differences of flow properties in a rock body (e.g. Lister and Williams, 1983). However, I prefer the term 'distribution' for this phenomenon and in this contribution I will use the term 'partitioning' to refer to differences in deformation rate between constituent phases at any point in a rock, in which the rock is treated as a continuum.

In this paper I present a simple approach for determining the flow properties of a composite rock from the properties of the constituent phases and the rock compositions on the basis of the notion of a multiphase continuum, (also termed superimposed continua). The multiphase continuum is a model of a heterogeneous medium that is composed of two or more phases. The forerunner of the notion of a multiphase continuum was a construction developed for continuum mechanics of a multicomponent material (Truesdell, 1957; Truesdell and Toupin, 1960; Truesdell, 1984). This construction considers multicomponent materials as continua in which several continua composed of their own component materials are spatially surperimposed on each other (e.g. Bowen, 1976; Truesdell, 1984). On the basis of a similar idea in continuum mechanics of multiphase material, multiphase materials are treated as a continua in which several continua composed of their own phase material are spatially superimposed on each other (e.g. Passman, 1977; Passman et al., 1984; Dobran, 1985; Dobran, 1991). Thus, despite the fact that the multiphase material is not continuous, it can nevertheless be treated as a continuum. In this approach multiphase properties are measured in terms of a compositional variable such as the volume fractions of constituent phases. Accordingly, details of the spatial distribution of constituent phases and other microstructural features such as grain-size and grain-shape are not considered. The present approach implicitly assumes that the rock is statistically isotropic. Any physical property of a multiphase continuum can be related to those of the constituent phases. The relationship may be derived from additive relationships for linear momentum, stress, and entropy production rate. It is assumed that both the flow behaviour of composite rocks and that of the constituent phases can be described by linear viscous constitutive relationships. The obtained results contain analytical relationships between the bulk rock viscosity (multiphase material as a whole), the volume fraction of their constituent phases, and the physical properties of the constituent phases. The relationships between the deformation rates of constituent phases and the deformation rate of the bulk rock, i.e. defor-

mation rate partitioning, is also derived. The obtained results are compared with published experiments and it is shown that the present simple approach satisfactorily accounts for the experimentally observed flow behaviours of two-phase rocks.

FLOW IN A MULTIPHASE CONTINUUM

Kinematics

Let B_α be a body composed of a phase α. For a multiphase material that consists of two or more phases, it is supposed that a sequence of bodies B_α, $\alpha = 1, 2, 3, \ldots$, occupy the same region of space simultaneously (Truesdell, 1984). Let X_α be a material point within a body B_α. The motion of B_α is the time sequential mapping of B_α onto a three-dimensional space as

$$x = x(X_\alpha, t),\tag{1}$$

where t is time. The velocity of phase α may be defined by

$$V_\alpha = \frac{\partial x(X_\alpha, t)}{\partial t}.\tag{2}$$

Since the total linear momentum of a multiphase material in a given region is the sum of the linear momenta of each phase,

$$\rho V = \sum \rho_\alpha V_\alpha,\tag{3}$$

where ρ is the bulk mass density of the multiphase material, ρ_α is the partial mass density of the phase α, and V is the mean velocity of the material. The bulk mass density is the sum of the partial mass densities of each phase,

$$\rho = \sum_\alpha \rho_\alpha,\tag{4}$$

where the partial mass density is the mass of phase α per unit volume of multiphase material.

$$\rho_\alpha = \phi_\alpha \gamma_\alpha,\tag{5}$$

in which γ_α is the mass of phase α per unit volume of phase α, and ϕ_α is the volume fraction of phase α.

Substituting equations (4) and (5) into equation (3) we have

$$V \sum \phi_\alpha \gamma_\alpha = \sum \phi_\alpha \gamma_\alpha V_\alpha.\tag{6}$$

In order to obtain the relationship between the velocity gradients for each phase and that for the bulk system, the gradient of the terms in equation (6) is taken and this yields

$$\left(\sum \phi_\alpha \gamma_\alpha\right) \nabla V = \sum \left[(\phi_\alpha \gamma_\alpha) \nabla V_\alpha + \nabla(\phi_\alpha \gamma_\alpha) \otimes u_\alpha\right],\tag{7}$$

where u_α is the velocity of phase α relative to the mean velocity,

$$u_\alpha = V_\alpha - V, \tag{8}$$

and the notation $\otimes$ means the tensor product. Similarly, the operation curl of terms in equation (6) give

$$\left(\sum \phi_\alpha \gamma_\alpha\right) \nabla \times V = \sum [(\phi_\alpha \gamma_\alpha) \nabla \times V_\alpha + \nabla(\phi_\alpha \gamma_\alpha) \times u_\alpha]. \tag{9}$$

Using the vorticity vector, equation (9) becomes

$$\left(\sum \phi_\alpha \gamma_\alpha\right)\omega = \sum [(\phi_\alpha \gamma_\alpha)\omega_\alpha + \nabla(\phi_\alpha \gamma_\alpha) \times u_\alpha], \tag{10}$$

where ω is the vorticity vector of bulk material, and ω_α is the vorticity vector of phase α. Following the usual practice, we can write the velocity gradient as the sum of the symmetric deformation rate tensor and the skew-symmetric spin tensor. Thus, equation (7) may be rewritten as

$$\left(\sum \phi_\alpha \gamma_\alpha\right)(D_{ij} + W_{ij})$$
$$= \sum \left[(\phi_\alpha \gamma_\alpha)(D_{ij}^\alpha + W_{ij}^\alpha) + \nabla(\phi_\alpha \gamma_\alpha) \otimes u_\alpha\right], \tag{11}$$

where D_{ij} is the deformation rate tensor of the bulk material, W_{ij} is the spin tensor of the bulk material, D_{ij}^α is the deformation rate tensor of phase α, and W_{ij}^α is the spin tensor of phase α.

Stresses and constitutive equations

Goodman and Cowin (1972) suggested the following constitutive relation for a granular material composed of solid particles and a vacuum:

$$T_{ij} = T_{ij}(\phi_0, \phi, \nabla\phi, \dot\phi, \theta, \nabla\theta, D_{ij}), \tag{12}$$

where T_{ij} is a stress tensor, ϕ is the volume fraction of the granular particles, ϕ_0, $\nabla\phi$, and $\dot\phi$ are, respectively, the initial value, the gradient measured in a reference frame fixed to the material, and a time derivative of the volume fraction of the particles, and θ is the temperature, $\nabla\theta$ is the thermal gradient, and D_{ij} is the deformation rate tensor. From thermodynamical considerations, these workers derived the following relationship,

$$T_{ij} - T_{ij}^0 = \xi\dot\phi\delta_{ij} + \lambda D_{kk}\delta_{ij} + 2\mu D_{ij}, \tag{13}$$

where T_{ij}^0 is the equilibrium stress, ξ is the linear coefficient for the time derivative of volume fraction of the granular particles, μ is the shear viscosity, λ is the second viscosity, and δ_{ij} is the Kronecker delta. Although equation (13) is derived for granular materials and there is no vacuum in my model, it can nevertheless be used as a linear constitutive equation for the two-phase material considered here, because there is formal

agreement in relation to the constitutive relation: both have only an independent volume fraction.

The stress T_{ij}^α acting on phase α can be defined at any point in the body. The stress T_{ij}^α is here referred to as a partial stress. Assuming each phase is linear viscous, the partial stress of phase α may be written as

$$T_{ij}^\alpha = -\phi_\alpha p_\alpha \delta_{ij} + \lambda_\alpha \phi_\alpha D_{kk}^\alpha \delta_{ij} + 2\phi_\alpha \mu_\alpha D_{ij}^\alpha, \tag{14}$$

where p_α is the partial pressure of phase α, D_{ij}^α is the deformation rate of phase α, μ_α is the shear viscosity of phase α, and λ_α is the second viscosity of phase α.

The total stress of a multiphase continuum, T_{ij}, may be related to the partial stress of each phase T_{ij}^α as

$$T_{ij} = \sum (T_{ij}^\alpha - \rho_\alpha u_{\alpha,i} u_{\alpha,j}), \tag{15}$$

where the second term in the right hand side is an apparent stress due to the transport of linear momentum (e.g. Passman, 1977; Nunziato and Walsh, 1980). In fact, the formulation of this relationship has been applied to multicomponent mixtures, motivated by the classical kinematic theory of gas mixtures (Truesdell, 1957; Truesdell and Toupin, 1960). This relationship may, therefore, be regarded as a fundamental assumption for the theory of continuum mechanics for multiphase materials.

For a two-phase material, using the relationship between total stress and partial stresses expressed by equation (15), the constitutive equation of a multiphase continuum (13), and the constitutive equation of each phase (14), the following relationship is obtained,

$$T_{ij}^0 + \xi\dot\phi_\alpha\delta_{ij} + \lambda D_{kk}\delta_{ij} + 2\mu D_{ij} =$$
$$\sum_{\alpha=1}^{2}(-\phi_\alpha p_\alpha\delta_{ij} + \lambda_\alpha\phi_\alpha D_{kk}^\alpha\delta_{ij}$$
$$+ 2\phi_\alpha\mu_\alpha D_{ij}^\alpha - \rho_\alpha u_{\alpha,i} u_{\alpha,j}). \tag{16}$$

Because the stress of a multiphase continuum in equilibrium should be the sum of the partial pressure of each phase, it follows that

$$T_{ij}^0 = -\sum (\phi_\alpha p_\alpha \delta_{ij}), \tag{17}$$

and equation (16) can be reduced to

$$\xi\dot\phi_\alpha\delta_{ij} + \lambda D_{kk}\delta_{ij} + 2\mu D_{ij}$$
$$= \sum_{\alpha=1}^{2}(\lambda_\alpha\phi_\alpha D_{kk}^\alpha\delta_{ij} + 2\phi_\alpha\mu_\alpha D_{ij}^\alpha - \rho_\alpha u_{\alpha,i} u_{\alpha,j}). \tag{18}$$

Entropy production rates

Let φ_α be the partial entropy production rate of phase α per unit volume of multiphase material. The total entropy production rate of the multiphase material φ is then, by the additive relation of the entropy

production rate,

$$\varphi = \sum \varphi_\alpha. \tag{19}$$

The entropy production rate per unit volume for viscous flow Ψ is

$$\Psi = \frac{T_{ij}^v D_{ij}}{\theta}, \tag{20}$$

where T_{ij}^v is the viscous stress tensor (e.g. de Groot and Mazur, 1962). The upper term in the right hand side of equation (20) is the dot product of T_{ij}^v and D_{ij}.

From equations (13) and (20), the entropy production rate of the multiphase material per unit volume, φ, is given as

$$\varphi = \frac{(\xi\dot\phi\delta_{ij} + \lambda D_{kk}\delta_{ij} + 2\mu D_{ij})D_{ij}}{\theta}. \tag{21}$$

From equations (14) and (20), the partial entropy production rate φ_α is given by

$$\varphi_\alpha = \frac{(\lambda_\alpha\phi_\alpha D_{kk}^\alpha\delta_{ij} + 2\phi_\alpha\mu_\alpha D_{ij}^\alpha)D_{ij}^\alpha}{\theta}. \tag{22}$$

Substituting equations (21) and (22) into equation (19), yields

$$(\xi\dot\phi_\alpha\delta_{ij} + \lambda D_{kk}\delta_{ij} + 2\mu D_{ij})D_{ij}$$
$$= \sum_{\alpha=1}^{2}(\lambda_\alpha\phi_\alpha D_{kk}^\alpha\delta_{ij} + 2\phi_\alpha\mu_\alpha D_{ij}^\alpha)D_{ij}^\alpha. \tag{23}$$

FLOW IN ROCKS AS A TWO-PHASE CONTINUUM

Rocks will now be considered as a two-phase continuum and the flow of these rocks will be considered on the basis of the above relations. The rock will be assumed to be composed of two solid phases, 1 and 2. The rock is also assumed to be fully dense, i.e. to have zero porosity, so that $\phi_1 + \phi_2 = 1$. The deformation is assumed to be incompressible, i.e. $D_{kk} = 0$. It is further assumed that the volume fraction of each phase remains constant during deformation, i.e. $\dot\phi_\alpha = 0$. Each phase is assumed to be uniformly distributed throughout the aggregate, i.e. $\nabla (\phi_\alpha\gamma_\alpha) = 0$. The apparent stress result from the transport of linear momentum owing to the relative velocity of each phase with respect to mean velocity is also assumed to be zero. This is because the flow in rocks can be regarded as slow flow, and the influence of apparent stress on the total stress is so small that it can be neglected.

Using these conditions, equation (11) becomes

$$(\phi_1\gamma_1 + \phi_2\gamma_2)[D_{ij} + W_{ij}] = (\phi_1\gamma_1)\left[D_{ij}^1 + W_{ij}^1\right]$$
$$+ (\phi_2\gamma_2)\left[D_{ij}^2 + W_{ij}^2\right]. \tag{24}$$

equation (10) becomes

$$(\phi_1\gamma_1 + \phi_2\gamma_2)\omega = (\phi_1\gamma_1)\omega_1 + (\phi_2\gamma_2)\omega_2. \tag{25}$$

equation (18) becomes

$$\mu D_{ij} = \phi_1\mu_1 D_{ij}^1 + \phi_2\mu_2 D_{ij}^2, \tag{26}$$

and equation (23) becomes

$$\mu D_{ij}D_{ij} = \phi_1\mu_1 D_{ij}^1 D_{ij}^1 + \phi_2\mu_2 D_{ij}^2 D_{ij}^2. \tag{27}$$

Substituting equation (25) into equation (24),

$$(\phi_1\gamma_1 + \phi_2\gamma_2)D_{ij} = (\phi_1\gamma_1)D_{ij}^1 + (\phi_2\gamma_2)D_{ij}^2, \tag{28}$$

where the following relationships have been used: $W_{ij} = e_{ijk}\omega_k$, $W_{ij}^\alpha = e_{ijk}\omega^\alpha{}_k$, and e_{ijk} is the permutation symbol.

Rearranging (26), (27) and (28), we have

$$(\phi_2\gamma_2^2\mu_1 + \phi_1\gamma_1^2\mu_2)\mu^2 - [\phi_1\phi_2(\gamma_1\mu_2 - \gamma_2\mu_1)^2$$
$$+ 2\mu_1\mu_2(\phi_1\gamma_1 + \phi_2\gamma_2)^2]\mu$$
$$+ (\phi_1\gamma_1 + \phi_2\gamma_2)^2(\phi_1\mu_1 + \phi_2\mu_2)\mu_1\mu_2 = 0. \tag{29}$$

$$D_{ij}^1 = \frac{[\phi_2\mu_2(\phi_1\gamma_1 + \phi_2\gamma_2) - \phi_2\gamma_2\mu](\phi_1\gamma_1 + \phi_2\gamma_2)}{([\phi_2\mu_2(\phi_1\gamma_1 + \phi_2\gamma_2) - \mu\phi_2\gamma_2]\phi_1\gamma_1}D_{ij}.$$
$$\qquad -[\phi_1\mu_1(\phi_1\gamma_1 + \phi_2\gamma_2) - \mu\phi_1\gamma_1]\phi_2\gamma_2)$$
$$\tag{30}$$

$$D_{ij}^2 = \frac{[\phi_1\gamma_1\mu - \phi_1\mu_1(\phi_1\gamma_1 + \phi_2\gamma_2)](\phi_1\gamma_1 + \phi_2\gamma_2)}{([\phi_2\mu_2(\phi_1\gamma_1 + \phi_2\gamma_2) - \mu\phi_2\gamma_2]\phi_1\gamma_1}D_{ij}.$$
$$\qquad -[\phi_1\mu_1(\phi_1\gamma_1 + \phi_2\gamma_2) - \mu\phi_1\gamma_1]\phi_2\gamma_2)$$
$$\tag{31}$$

Equation (29) relates the bulk rock viscosity μ to μ_1, μ_2, ϕ_1, γ_1, and γ_2. Equations (30) and (31) relate the deformation rate of each phase to the deformation rate of the bulk rock.

Bulk rock viscosity–volume fraction relationship

Equation (29) may further be simplified using the density contrast, $a = \gamma_2/\gamma_1$, the viscosity contrast, $b = \mu_2/\mu_1$, and the normalized bulk rock viscosity $\mu^* = \mu/\mu_1$, as

$$[b\phi_1 + a^2(1 - \phi_1)\mu^{*2}]$$
$$- \left\{\phi_1(1 - \phi_1)(b - a)^2 + 2b\phi_1 + [a(1 - \phi_1)]^2\right\}\mu^*$$
$$+ b[\phi_1 + a(1 - \phi_1)]^2[b(1 - \phi_1) + \phi_1] = 0. \tag{32}$$

We assume $\mu_1 < \mu_2$, hence $b > 1$. Solutions to this quadratic equation are

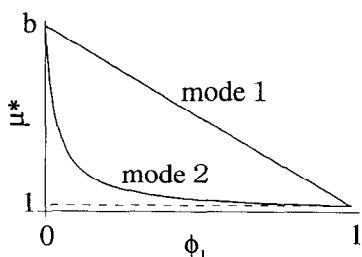$$\mu^* = (1 - b)\phi_1 + b, \tag{33}$$

and

Fig. 1. Schematic curves showing the relation between the normalized bulk rock viscosity ($\mu^* = \mu/\mu_1$) and the volume fraction of less viscous phase (ϕ_1) predicted by the model. Mode 1 behaviour shows a linear relationship, mode 2 behaviour shows a non-linear relationship. b is the viscosity contrast between the phases (μ_2/μ_1).

$$\mu^* = \frac{\left[a^2 - 2a(a-1)\phi_1 + (a-1)^2\phi_1^2\right]b}{a^2 + (b-a^2)\phi_1}. \quad (34)$$

The solution given by equation (33) yields a linear relationship between μ^* and ϕ_1 (Fig. 1), and indicates that μ^* is independent of the density contrast behaviour of the two phases. This relationship is named mode 1 behaviour. The solution given by equation (34) implies that providing the $a \neq b$ relationship, μ^* vs ϕ_1, is non-linear and dependent on both of the density and viscosity contrasts (Figs 1 & 2). This is named mode 2 behaviour. Where $a = b$, equation (34) is reduced to equation (33). In mode 2 behaviour, the rate of decrease of the bulk rock viscosity decreases as the density contrast increases (Fig. 2a). The rate of decrease increases with increase of the viscosity contrasts (Fig. 2b).

Deformation-rate of constituent phases

Equations (30) and (31) relate the deformation rate of each phase to the deformation rate of the bulk rock. The coefficients of D_{ij} in these equations is designated the deformation rate partition coefficients of phase 1 and 2, P_1 and P_2, respectively. Thus $D_{ij}{}^1 = P_1 D_{ij}$, $D_{ij}{}^2 = P_2 D_{ij}$. Using the density contrast, a, the viscosity contrast, b, and the normalized bulk rock viscosity, μ^*, the deformation rate partition coefficients P_1 and P_2 may be expressed as

$$P_1 = \frac{[b(\phi_1 + a(1 - \phi_1)) - a\mu^*](\phi_1 + a(1 - \phi_1))}{([b(\phi_1 + a(1 - \phi_1)) - a\mu^*]\phi_1 \\ -[\phi_1(\phi_1 + a(1 - \phi_1)) - \mu^*\phi_1]a)}, \quad (35)$$

and

$$P_2 = \frac{[\mu^* - (\phi_1 + a(1 - \phi_1))](\phi_1 + a(1 - \phi_1))}{([b(1 - \phi_1)(\phi_1 + a(1 - \phi_1)) - a\mu^*(1 - \phi_1)] \\ -[(\phi_1 + a(1 - \phi_1)) - \mu^*](1 - \phi_1)a)}. \quad (36)$$

As shown above there are two relationships between μ^* and ϕ_1, i.e. mode 1 and mode 2. For mode 1 beha-

viour, substituting equation (33) into equations (35) and (36), we have

$$P_1 = P_2 = 1; \quad (37)$$

i.e. the deformation rate of each phase are equal to that of the bulk rock. For mode 2 behaviour, substituting equation (34) into equations (35) and (36), the partition coefficients become

$$P_1 = \frac{b(a + \phi_1 - a\phi_1)}{a^2 + (b - a^2)\phi_1}, \quad (38)$$

$$P_2 = \frac{a(a + \phi_1 - a\phi_1)}{a^2 + (b - a^2)\phi_1}, \quad (39)$$

i.e. the deformation rates of each phase are dependent on the density contrast, the viscosity contrast, and the volume fraction ϕ_1. In mode 2 behaviour the ratio of the two deformation rate partition coefficients P_1 and P_2 is equal to the ratio of the viscosity contrast and the density contrast,

$$\frac{P_1}{P_2} = \frac{b}{a}. \quad (40)$$

Thus, if the viscosity contrast is greater than the density contrast (regime 1 in Fig. 3), the deformation rate of the less viscous phase is larger than that of the more viscous phase. Alternatively, if the viscosity contrast is smaller than the density contrast (regime 2 in Fig. 3), the deformation rate of the less viscous phase is smaller than that of the more viscous phase. If
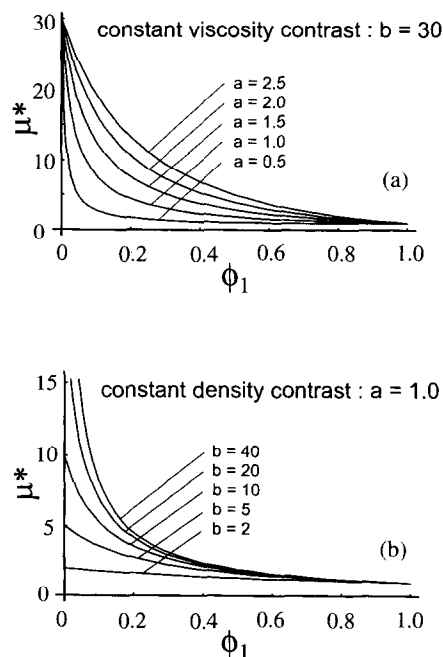

Fig. 2. The relationship between the normalized bulk rock viscosity (μ^*) and the volume fraction of the less viscous phase (ϕ_1) calculated from equation (33): (a) for different density contrasts ($a = \gamma_2/\gamma_1$) and (b) for different viscosity contrasts.
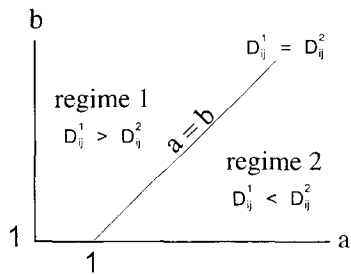
Fig. 3. Two regimes of flow behaviour defined by the relative deformation rates of the constituent phases, as plotted in viscosity contrast (b)–density contrast (a) space. In regime 1 ($b > a$) the deformation rate of the less viscous phase $D_{ij}{}^1$ is larger than that of the more viscous phase $D_{ij}{}^2$, while in regime 2 ($b < a$) the deformation rate of the less viscous phase is smaller than that of the more viscous phase. When $a = b$, the deformation rates of the two phases are equal to each other.

$a = b$, the deformation rate of the less viscous phase is consistent with that of the more viscous phase, thus, in this case mode 1 only occurs.

The deformation rate partition coefficients for each phase are shown as a function of density contrast in Fig. 4. The two regimes of deformation rate partitioning delimited in a–b space (Fig. 3) correspond to the two types of mode 2 behaviour. In regime 1 ($a < b$) the partition coefficient for the less viscous phase is greater than 1: i.e. the deformation is partitioned into the less viscous (weak) phase. Increasing the volume fraction of the less viscous phase decreases the partitioning of deformation into the less viscous phase (Fig. 5). In regime 2 ($a < b$), the partition coefficient for the less viscous phase is always less than 1. Increasing the volume fraction of the less viscous

phase increases the deformation rate of the less viscous phase relative to that of the more viscous phase (Fig. 5).

The deformation rate partition coefficients for each phase are shown for different values of viscosity contrast as a function of volume fraction (Fig. 6) and density contrast (Fig. 7). It is seen that in the range of $\phi_1 > 0.2$ or of $a < 3$, the partition coefficient is insensitive to the viscosity contrast provided this contrast is greater than about 20–40, and is determined primarily by the volume fraction and density contrast.

DISCUSSION

Comparison with laboratory data for flow strength of two-phase rocks

There are some experimental data on deformation of two-phase rocks that may be relevant to the theoretical model derived above. Jordan (1987) conducted a series of deformation experiments using synthetic calcite–halite rocks at temperatures up to $200°C$, confining pressures of between 150 MPa and 400 MPa, and strain-rates of between 10^{-4} s^{-1} and 10^{-5} s^{-1}, and observed a rapid decrease in strength as the volume fraction of the less viscous phase, halite, was increased. A similar result was obtained by Bloomfield and Covey-Crump (1993) for the same system. The results obtained by Price (1982) and Ross et al. (1987) for anhydrite–halite systems were also similar to those obtained above.
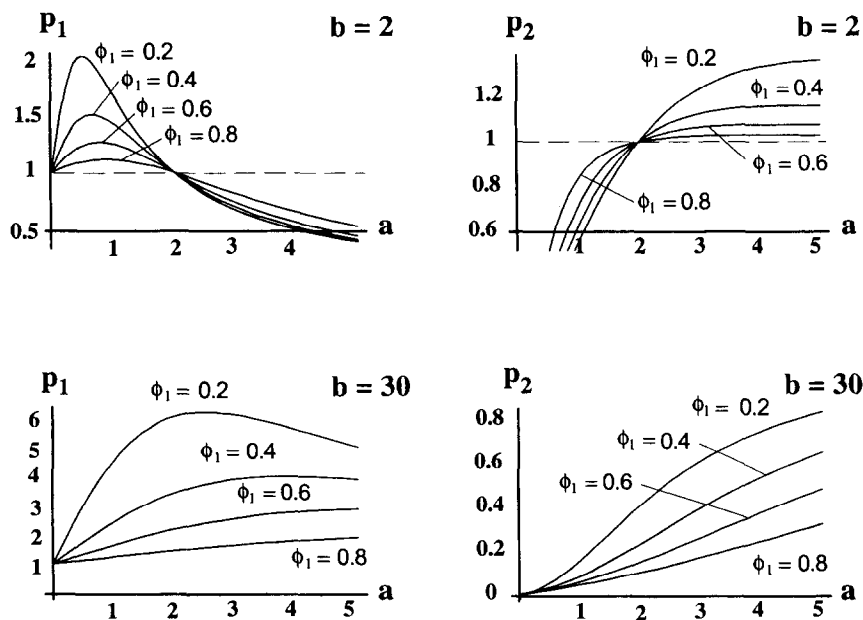

Fig. 4. The relationship between the deformation rate partition coefficients of the constituent phases, P_1, P_2 and the density contrast, a, as a function of the volume fraction of the less viscous phase, ϕ_1, for different viscosity contrasts, b.
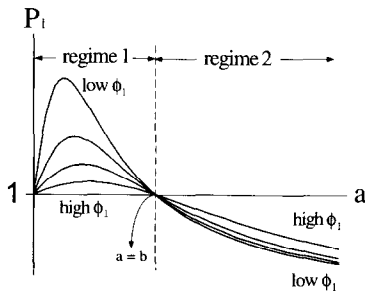
Fig. 5. A schematic diagram showing two regimes of flow behaviour in the less viscous phase deformation rate partition coefficient-density contrast space. In regime 1 the viscosity contrast is larger than the density contrast and the deformation is partitioned into the less viscous phase. In regime 2 the viscosity contrast is smaller than the density contrast and the deformation is partitioned into the more viscous phase. The influence of the volume fraction of the less viscous phase (ϕ_1) on the behaviour is also illustrated.

In these experiments, one phase (halite) was deformed by intracrystalline plasticity but the other, stronger phase, (calcite or anhydrite) was semibrittle. Hence, the flow behaviour of the bulk rock and of each phase may significantly depart from Newtonian. To a first approximation, however, the notion of the equivalent viscosity η that can be defined as $\eta = \sigma/3\dot{\varepsilon}$, where σ is the differential stress supported by the specimen at strain-rate $\dot{\varepsilon}$ (Griggs, 1939; Carter and Heard, 1970), can be used. The equivalent viscosity η is deduced from their experiments and is compared with the present model. Then the viscosity contrast is

replaced by the equivalent viscosity contrast. Figure 8 shows the experimental results of Jordan (1987, 1988) and Price (1982) in comparison with the results estimated from the model. The equivalent viscosity contrast in those experiments is estimated to be 6 for the experiments of Jordan (1987) and Price (1982), and 7 for the experiment of Jordan (1988). The density contrast for calcite and halite is 1.3 and that for anhydrite and halite is 1.1 (Holland and Powell, 1990). Despite the simplifications and the assumptions made, mode 2 behaviour of the theoretical results are in good agreement with the experiments.

Comparison with the behaviour of strain partitioning obtained by Bloomfield and Covey-Crump (1993)

As mentioned above, the relationship between the flow strength and the volume fraction in the two-phase aggregates used in the experiment of Bloomfield and Covey-Crump (1993) is similar to the results obtained by Price (1982), Jordan (1987, 1988) and Ross et al. (1987), i.e. the behaviour is consistent with the mode 2 behaviour. Moreover, Bloomfield and Covey-Crump (1993) estimated the strain partitioning between the phases during the experimental deformation of a two-phase aggregate composed of calcite and halite using both mechanical and microstructural methods. For the mechanical method, the strain of both halite and calcite in the aggregate were determined by two additive relations of the flow stress and the strain using the
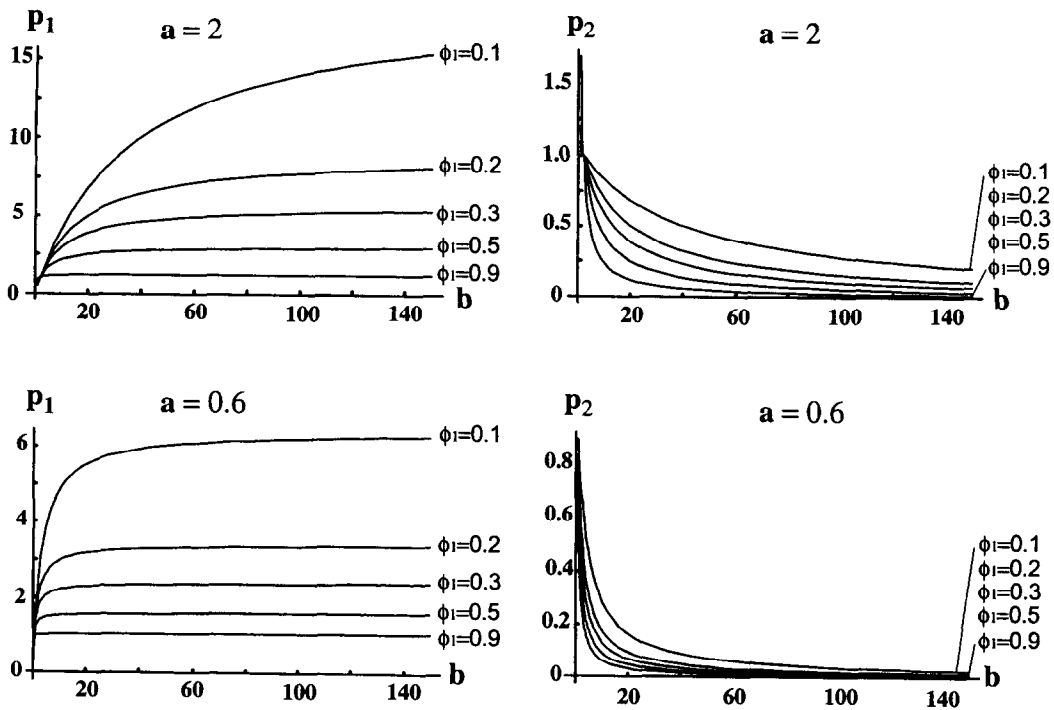

Fig. 6. The relationships between the deformation rate partition coefficients of the constituent phases and the viscosity contrast (b) for different volume fractions at various density contrasts (a).
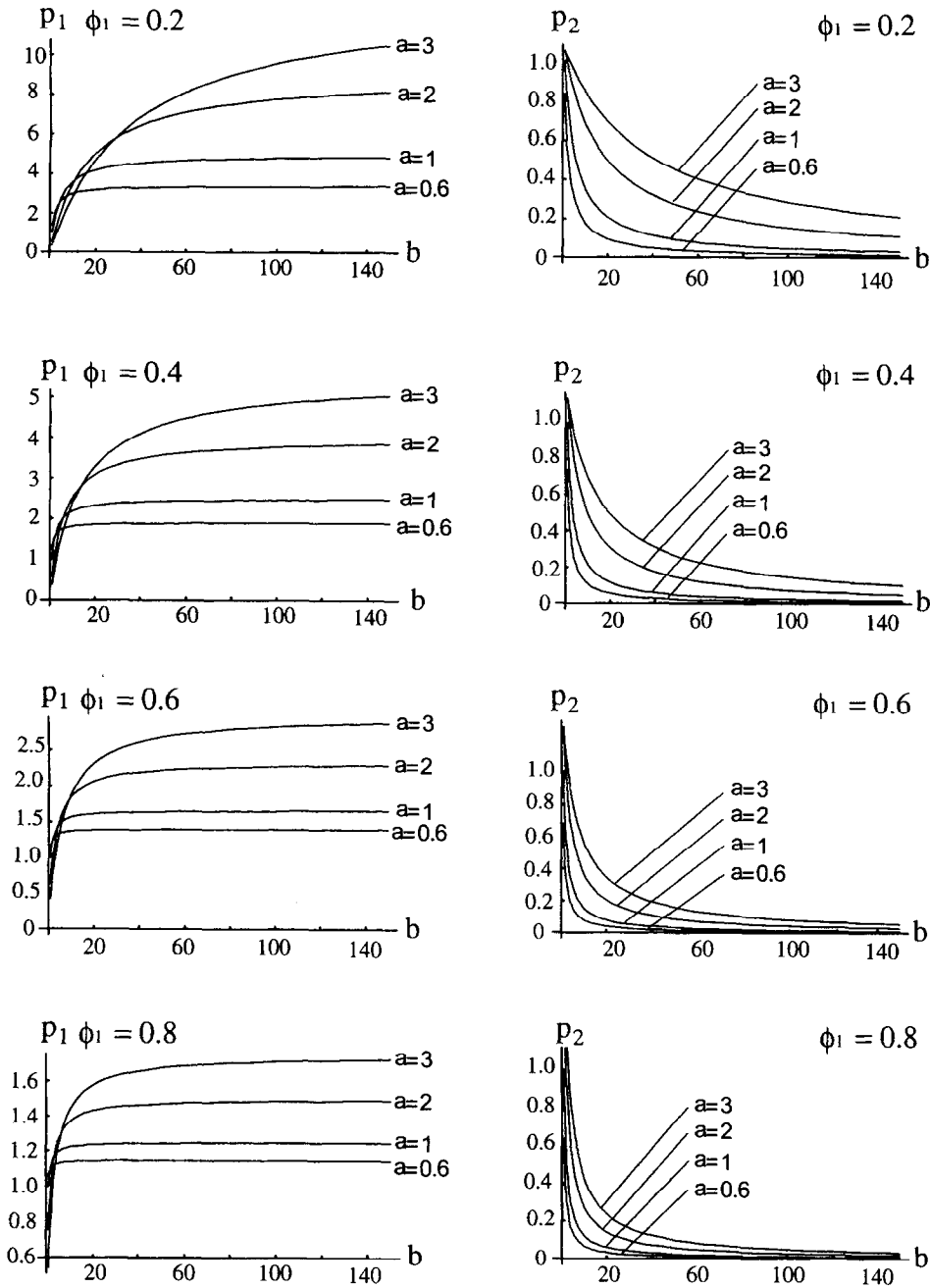
Fig. 7. The relationships between the deformation rate partition coefficients of the constituent phases and the viscosity contrast (b) for different density contrasts (a) with various volume fractions of the less viscous phase.

stress–strain curves of the aggregates and single-phase end members, halite and calcite. The mechanical analysis suggests that there is a strain partitioning into the less viscous phase, halite, and that there is a decrease of strain partitioning into less viscous phase as its volume fraction increases. In contrast, the microstructural analysis suggests that there is no strain partitioning between the two phases. The result they obtained from the mechanical analysis is in accordance with the mode 2a behaviour of our model.

The mode of the partitioning of deformation rate in nature

The bulk flow in rocks must be related to the flow of its constituent phases. As described above, the model of multiphase continuum developed here delimits two modes of flow behaviour according to whether there is no deformation rate partitioning between the phases (mode 1 behaviour) or there is deformation rate partitioning (mode 2 behaviour). From
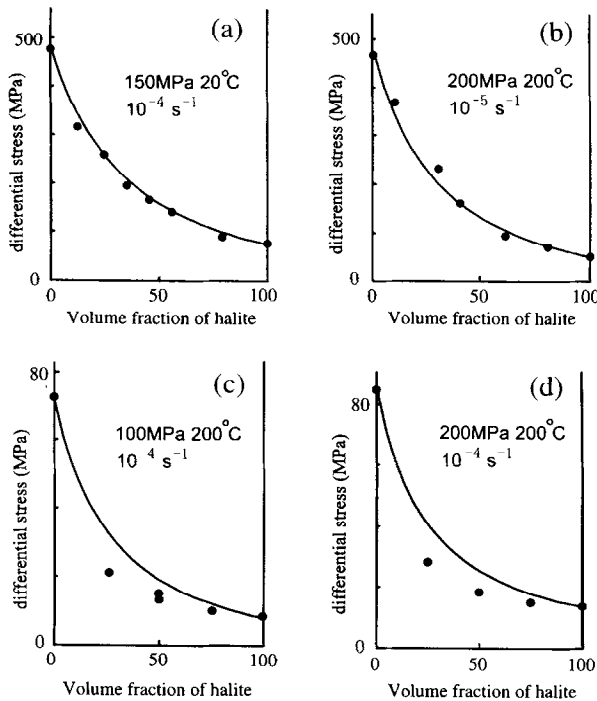
Fig. 8. Results of experimental investigations on two phase aggregates compared with the predictions of the model. The solid lines show predictions of the model. The solid circles show results of experiment. (a) Model for $a = 1.3$ and $b = 6$. Experimental data from Jordan (1987), calcite-halite aggregates at 20 C, 150 MPa confining pressure and strain rate of 10^{-4} s^{-1}. (b) Model for $a = 1.3$ and $b = 7$. Experimental data from Jordan (1988), calcite–halite aggregates at 200 C. 200 MPa confining pressure and strain rate of 10^{-5} s^{-1}. (c) Model for $a = 1.1$ and $b = 6$. Experimental data from Price (1982), anhydrite–halite aggregates at 200 C, 100 MPa confining pressure and strain rate of 10^{-4} s^{-1}. (d) Model for $a = 1.1$ and $b = 6$. Experimental data from Price (1982), anhydrite-halite aggregates at 200 C. 200 MPa confining pressure and strain rate of 10^{-4} s^{-1}.

comparison with experimental results mode 2 behaviour is more plausible than mode 1 behaviour in nature. Mode 2 behaviour is further subdivided into two regimes according to whether there is strain partitioning into the less viscous phase (mode 2a behaviour), or the strain is partitioned into the more viscous phase (mode 2b behaviour). In other words, the less viscous phase is more ductile in mode 2a while it becomes less ductile in mode 2b behaviour. A critical parameter determining which mode occurs is the ratio of the density contrast to the viscosity contrast. Density contrasts of common rock-forming minerals are usually close to unity. For example, albite/quartz $\approx 1.0 \sim 1.1$, anorthite/quartz $\approx 1.0 \sim 1.1$, dolomite/calcite $\approx 1.0 \sim 1.1$, enstatite/forsterite $\approx 0.9 \sim 1.0$, diopside/forsterite $\approx 0.9 \sim 1.0$ (Holland and Powell, 1990). On the other hand, viscosity contrasts are more variable, probably ranging over several orders of magnitude, and therefore, it is expected that the viscosity contrast is usually greater than the density contrast. Therefore, mode 2a behaviour should dominate in nature. However, theory predicts that mode 2b should

also occur if the viscosity contrast is very small and is less than the density contrast.

Strain refraction

The distribution of strain (or deformation rate) in multilayer rocks in which layers have different rheologies is called strain refraction (e.g. Treagus, 1988). A rule of strain refraction between two layers both of which have Newtonian rheology is given by

$$\frac{\dot{\varepsilon}_i}{\dot{\varepsilon}_j} = \frac{\mu_j}{\mu_i},$$

where $\dot{\varepsilon}_i$ is the layer-parallel shear deformation rate in layer i with viscosity μ_i, $\dot{\varepsilon}_j$ is the layer-parallel shear deformation rate in layer j with viscosity μ_j (Cobbold, 1983; Treagus, 1983, 1988, 1993; Weijermars, 1992). Thus the ratio of the layer-parallel shear deformation rates is equal to the inverse viscosity contrast. For two-phase rocks, in which phases are distributed homogeneously, the ratio of the deformation rate among phases is given by equation (40). When the density contrast between two phases in rocks is unity, the ratio of deformation rates is equal to the inverse viscosity contrast. It is evident that the rule of strain refraction obtained for the multilayer rocks is consistent with the result obtained for multiphase rocks. A difference between multiphase rocks and multilayer rocks is that the relationship for multiphase rocks obtained in this paper is independent of spatial direction, while the rule of strain refraction obtained for multilayer rocks is only valid in layer-parallel directions.

Other remarks on the present approach

The present approach provides a simple model for the flow behaviour of two phase rocks. It should be emphasized again that the volume fraction is the only state variable considered for describing the structure of rocks and that the other microstructural features, such as grain-size, grain-shape or grain boundary geometry, the crystal lattice preferred orientation and other structural anisotropy, are not incorporated in the model. In other words, the flow behaviour of the two phase rock is only dependent on the volume fraction of the two phases and is independent of microstructures. However, natural rocks are highly variable in structure and it is quite plausible that these microstructures affect the flow behaviour in rocks, which has been an important subject in structural geology (e.g. Handy, 1990, 1992; Tullis et al., 1991; Dell'Angelo and Tullis, 1996). The present model, therefore, clearly has a limitation and needs to be treated with caution. Nevertheless, it has value as a reference model to which the behaviour of natural rocks can be compared. It is hoped that the present model can be developed and improved, so as to incorporate any of the

above mentioned structural features. For this purpose, appropriate state variables that express these structural variables will need to be formulated.

CONCLUSIONS

In this paper the flow of polymineralic rocks is considered on the basis of the notion of a multiphase continuum. In many previous attempts of formulating flow properties of two-phase rocks, the manner of partitioning of deformation rate between constituent phases was assumed either implicitly or explicitly. In the present treatment both the viscosity of two-phase aggregates and the partitioning of deformation rate are determined as functions of the volume fraction and mechanical properties of constituent phases, assuming linear viscous rheologies. Two modes of behaviour are theoretically identified: a linear relation (mode 1), and non-linear relation (mode 2). The non-linear relation may be further subdivided into mode 2a and mode 2b on the basis of partitioning of deformation rate for each constituent phase. Mode 2a behaviour shows good agreement with published deformation experiments of synthetic two-phase rocks.

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